Physics 200-04

Ptolemy's Planets

Ptolomey developed a system for the description of the planets in the 2nd century AD. He was Greek, born in Egypt (hence his name), and he extended and codified the immense work that Hiparcus had done in the 2nd century BC in gathering together the observations of the Babylonians and Greeks for the previous millenium.

His structure for the motion of the planets was extremely sophisticated and anticipated much of the work of Kepler in a form more than adequate for the observational accuracy of his day.

Ptolomy was immensely practical, and his purpose was not to give a grand theoretical structure to the universe, but rather to the describe in a consise mathematical form the world as it is. Thus, since he was describing the motion of the planets in the night sky of the earth, the earth formed for him the center of the description. It is the planets that are seen to move, and his purpose was to describe that motion. The motion was two-fold. The planets moved in a great circle roughly around the earth. In addition, each of the planets executed a secondary motion around another circle, called the epicycle, around the point on the first circle which moved around the earth.

The planets could be divided into two lots. There were those which could at appropriate times of the year be seen almost anywhere along the celestial equator (the ones we now call the outer planets– Mars, Jupiter and Saturn) and those who never were seen very far above the horizon at night (Mercury and Venus). In addition there was the Moon and the Sun.

It was already known that the earth was a sphere, and that furthermore the size of that sphere had been roughly measured by Aristarchus to within about 10% of the correct value. It was also recognized that the eclipses of the moon were caused by the earth casting its shadow on the moon. Knowing the size of the earth and measuring the curvature of the shadow of the earth on the moon, it was known that the moon was about 100 times as far away as the radius of the earth. Furthermore, the other planets, since they were at various times eclipsed by the moon, were know to be further away than the moon. There was however no way of determining how far away they actually were. Thus in any description of the planets, there was always an unknown factor, namely the size of the orbits, by which the orbits could be scaled.

For each of the planets, except the sun, there was a two fold motion. The first was the motion of the center of the epicycle around the earth. This was not simply circular motion. The center of the circle was not at the earth, it was displaced from the earth by a varying amount, varying for each of the planets. Furthermore, the motion around the circle was not uniform. Rather the angular motion about the circle was uniform about a point equidistant from the center of the circle to the earth's distance from the center, and along the same line. It was around this point, the equant, that the center of the epicycle seemed to move uniformly. The epicycle on the other hand was assumed to be a circle around which the planet moved uniformly.

In this structure of earth-center-equant, Ptolomy anticipated two of Kepler's laws. The earth and the equant are the two foci of Kepler's ellipse. To lowest order in the eccentricity (ratio of the distance of the earth to the orbit's center over the radius of the orbit), the equal rotation of the planet about the equant is precisely Kepler's second law– that the planet's radius (in this case the line from the earth to the center of the epicycle) traces out equal areas in equal times.

It was almost certainly not the epicycle-main cycle that Alphosos ** of Spain complained about in the 11th century ADi [If God had asked me, I would have suggested something simpler] , but rather this complicated earth-center-equant construction.

As stated there was no scale to this construction. If one scaled the main cycle and the epicycle by the same amount, then the observations of the motion of the planet against the night sky were unchanged (assuming one kept the radii much larger than the moon's distance).

There was however a very strange feature of this construction. For each of the outer planets, the period of revolution along the epicycle was exactly one year. For each of the inner planets, the revolution about the main cycle was exactly one year. Furthermore, if one drew the main cycle/epicycle, one found that each of the planets AND the sun were always at the same place along the epicycle(for outer) and main cycle (for inner) as was the Sun.

It was Copernicus who realised how to explain this. His first task was to ignore Ptolomy's earth-center-equant construction and just assume that all of the main cycles simply went around the earth in a circle. His second was to realise that the epicycle and the main cycle could be interchanged. Ie, the motion of the planet would be exactly the same if he centered the epicycle on the earth and had the main cycle of the outer planets orbit around a point on the epicycle. For the outer planets, this made the new epicycle (former main cycle) much larger thant the epicycle (9 times as large for Saturn), and made

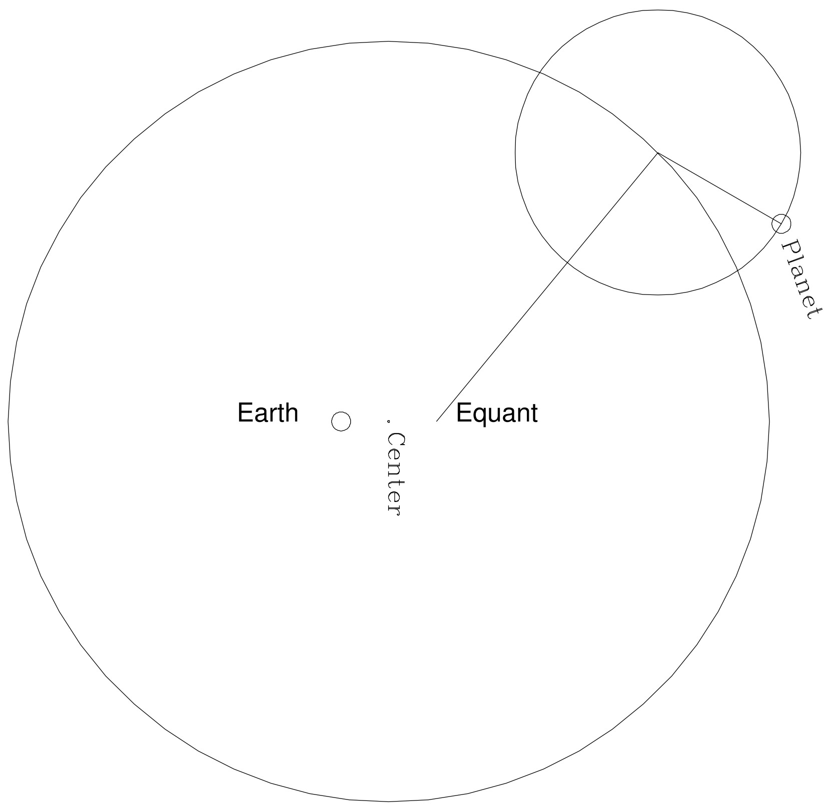

Figure 1: The structure of Ptolemy's orbit for one of the planets. The sun had no epicycle, while the moon's motion was very complex. For the outer planets, the eqicycle period was a year, while for the inner planets, the main cycle period was a year.

the construction look lopsided. However, now all of the new main cycles of the planets were such that the center of the epicycle orbited the earth once a year, just as the sun did. Furthermore, each of the centers of the new epicycles was in exactly the same direction from the earth as was the sun. If one now adjusted the scale of each of the orbits so that these new main cycles had the same radius as had the orbit for the sun, then the centers of all of the epicycles fell onto the sun. Ie, if one choose this scale for each of the planets, then the planets looked as though they were all orbiting around the sun, and the sun then orbited around the earth.

Alternatively, Copernicus realised that he could also change his viewpoint and regard the sun as stationary, and that then the earth would have to revolve around the sun instead, and become exactly the same as the planets. The moon, that most complicated of objects in the sky, now would have to revolve around the earth which revolved around the sun, making its motion even more complex than it had been before.

Furthermore, because in the Ptolemeic system, the epicycles of the outer planets had been simple circles centered at that moving point along the main cycle if he kept Ptolomy's earth-center-equant construction, you had a discrepancy between the motion around the new main cycle (old epicycle) of the outer planets, and the motion of the sun and venus around their main cycles, and his wonderful concordance of all of the main cycles and the sun would fall apart. For this and other philosophical reasons, Copernicus rejected this aspect of Ptolomy and insisted that the circle being the most perfect figure, and uniform circular motion the most perfect motion, these must be the basis of all planetary motion. But this meant that his system was not in agreement with the measured motion of the planets. He then revived the idea of epicycles (circles with uniform motion around the epicycle) to try to refit the data to his model. These new Copernican epicycles had the advantage of being much smaller in general than the Ptolomeic epicycles had been and furthermore had periods which were multiples of the planetary period around the sun, rather than all having the same period. But, in terms of beauty there was little to choose between the two.

The Copernican was also in conflict with experience. If the earth were whirling at the stupendous speeds required around the sun, surely there would be evidence thereof. All of the mechanical theories of the time asserted that the motion of bodies was toward a state of rest. Thus, if you dropped a canon ball from a high tower, it would go toward a state of rest. The

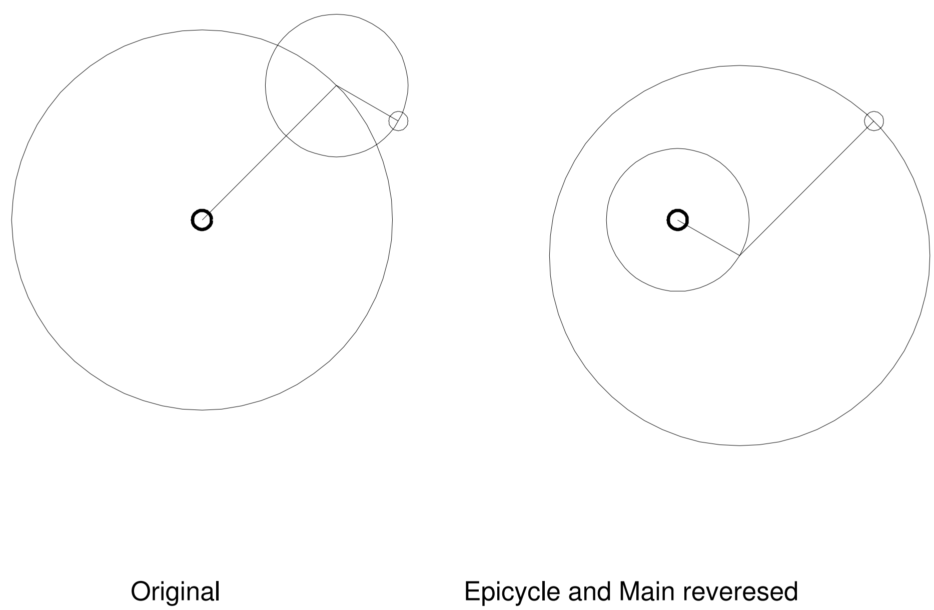

Figure 2: The path of the outer planets through the night sky remains the same if we exchange the role of the epicycle and the main cycle. This is strictly true only if we ascribe the center-equant structure to the new epicycle. Copernicus ignored this feature of Ptolemy's system.

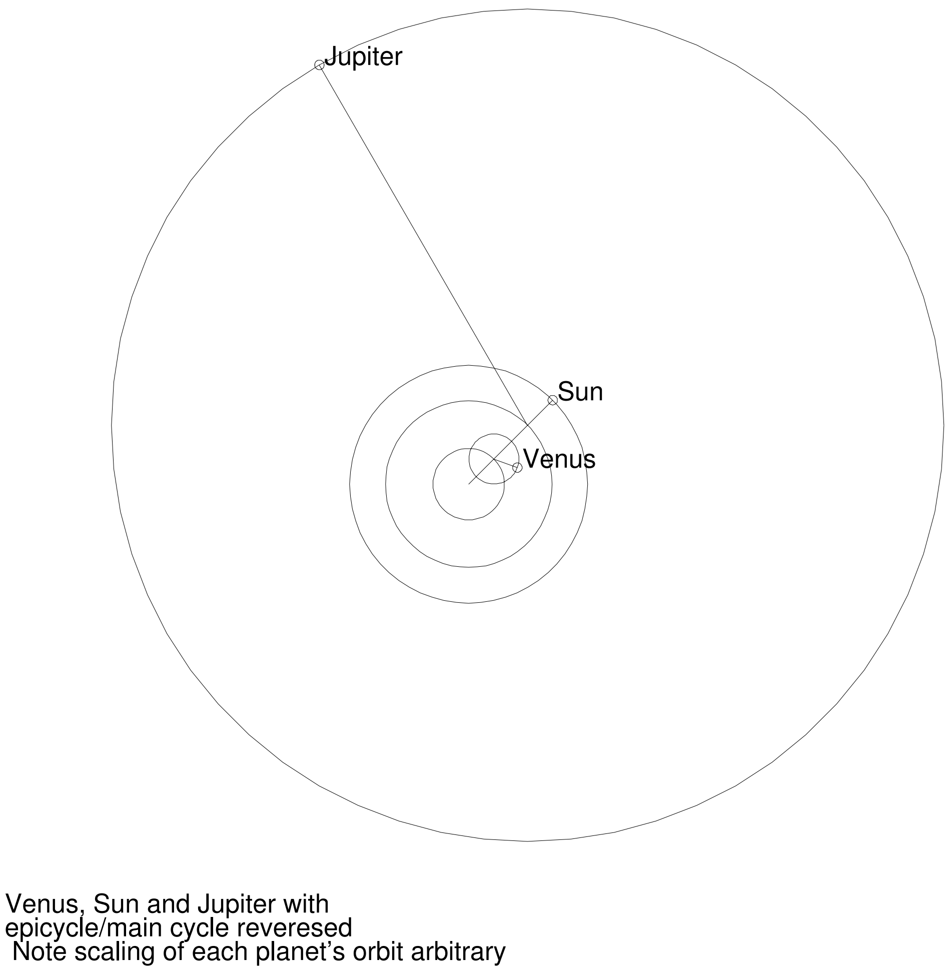

Figure 3: Once the main/epicycle were switched for the outer planets, then the center of the new epicycle on the orbit of each of the planets lined up with the sun throughout the year. By rescalling the orbits, the centers of all the epicycles of all the planets could be made to coincide with the sun.

earth would whizz around at its stupendous speed, and leave the canon ball behind. And this did not happen. And why would not the air get left behind as the earth moved at its tremendous velocity.

Futhermore, the stars were clearly much further away than the planets. If the earth moved, one would expect to see the stars move, due to parallax, as the earth moved around the sun. The only way they would not would be if they were much much much further away than Saturn was (which was nine times as far away as the sun was already). This was clearly an absurd conclusion. How could the world be that large. Besides, this Copernican theory now placed into the sky two different types of object. The planets, and the moon. The planets went around the sun, while the moon still circled the earth. Clearly this was an unwarranted complexity in the perfection of the heavens.

Tycho Brahe, the greatest observational astronomer since Hiparcus, rejected the notion that the earth moved, and instead accepted the intermediate point in Copernicus's journey, namely that the planets orbited the sun, while the sun and all the planets orbited the earth. This kept the earth at rest (obviating the problem with the parallax of the stars), but clearly kept a rather complex set of motions (the moon and sun orbit the earth and all the rest of the objects orbit the sun).

Kepler, who despite his employer, Brahe's, ideas, clung to the Copenican viewpoint. However, he essentially had to throw out Copernicus's circles, and go back to Ptolemy, and reinstitute the,now sun-center-equant construction. Brahe had given him Mars to understand, the planet, other than the essentially unobservable Mercury with the largest eccentricity. It was the discrepancy between Brahe's observations of Mars with even this model that led him to replace Ptolomy's circle centered at the midpoint between equant and sun, with an ellipse with the Sun at one focus and the equant the other focus. This very slighly distored the shape of the Mar's orbit from Ptolomy's circle. Furthermore he found that his second law, that the area swept out by the line from the sun to the planet was always the same in the same time period, again gave better agreement with the location of Mars in its orbit than did Ptolomy's uniform motion around the equant. His third law would have been impossible for Ptolomy to have found since it relates the period of the orbit (something well known to Ptolomy) with the radius of the orbit, something only set by Copernicus's work.

This history is brilliantly described in the book, "The Discovery of Dynamics" by Julian Barbour (Oxford U Press, 1999) and I strongly encourage anyone interested to read his far more detailed and fascination account.