Physics 200-04
Quantum Prehistory

Bohr Sommerfeld quantum rules

As an example of the quantization conditions which rose out of the Bohr-sommerfeld program, consider the derivation of the Energy levels of the H-like atom, but with many charges in the nucleus.

The Bohr-Sommerfeld program basically said that the mechanics was classical mechanics, but that only those solutions of the classical system which obeyed certain quantization conditions were valid. These were such that for a degree of freedom of the system, the integral

$$\int p_i v_i dt = nh \tag{1}$$

where p_i is the momentum of the i^{th} degree of freedom and v_i is the velocity of that degree of freedom. The integral is to be taken over a time period in which the arguments repeat themselves. In the simplest case this was one orbit, but in the more general case various momenta repeated themselves with different periods. It was unclear exactly what constituted a degree of freedom but this periodicity requirement tended to pick out certain components of the momentum.

Let us apply this to H-like atoms with Z positive charges in the nucleus and one electron orbiting that nucleus.

Let us assume that the electron is orbiting in a circular orbit of radius R. Then, we have (equating the centripidal acceleration to the force of the cental charge on the electron

$$\frac{mv^2}{R} = \frac{Ze^2}{4\pi\epsilon_0 R^2} \tag{2}$$

The quantization condition is

$$nh = \int pvdt = mv^2 T \tag{3}$$

where T is the period of the orbit. But we also have

$$v = 2\pi R/T \tag{4}$$

We can rewrite the force balance equation as

$$\frac{Ze^2}{4\pi\epsilon_0} = mv^2 R = \frac{mv^3 T}{2\pi} = nh\frac{v}{2\pi} \tag{5}$$

or

$$v = \frac{Ze^2}{2\epsilon_0 nh} \tag{6}$$

The energy of the electron is

$$E = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\epsilon_0 R} = -\frac{1}{2}mv^2$$
$$= -\frac{mZ^2e^4}{8\epsilon_0^2 h^2}\frac{1}{n^2} \tag{7}$$

Comparing to the Bohr formula derived from the Balmer/Rydberg formula we have

$$R = \frac{mZ^2e^4}{8c\epsilon_0^2 h^3} \tag{8}$$

Ie, the Rydberg constant, measured from the spectra of Hydrogen could be derived from the elementary constants , m, c, h, ϵ_0, e.